

This paper isn't finished, but there should be enough information here to get you started.

By: Iowa Hills Software, IowaHills.com
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The Design of Direct Coupled Band Pass Filters

In February 1957, Seymour Cohn published "Direct Coupled Resonator Filters" in Proceedings of the IRE. Since then, a very large number of articles have been published giving examples of these filters.

This paper breaks this design method into three parts: the matching network, the resonator, and the impedance inverter. It describes the design process for each of these sections in some detail.

There are two important aspects to this design method that make it so powerful, and that you should remember as you read through this paper. First, you will see that there are many possible topologies for the resonator, impedance inverter, and matching network sections. Remember that, within reason, any of these topologies can be mixed with any other section topology.

Second, the goal of this methodology is to allow the designer to arbitrarily set one of the two component values in the resonator circuit, and then calculate the other value based on the inverter and matching circuits. For example, if a parallel LC is to be used as the resonator, we can first choose a convenient value for the inductor, and then design the inverter and matching networks which will determine the value of the capacitor.

The ability to do this is important because it is the availability of inductors (both their Q and nominal value) that typically drives a design.

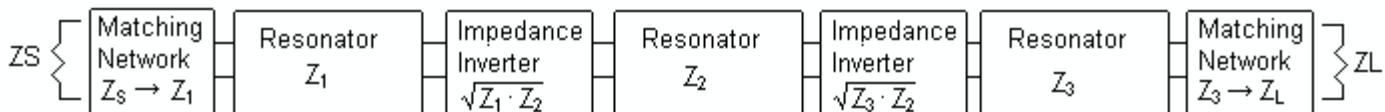
The Direct Coupled Topology

Each resonator section has a characteristic impedance Z_R , which is typically determined by the inductor value we choose, so it can take on a wide range of values. Each of the resonators can, and typically will, have a different characteristic impedance.

The filter is terminated by a source and load impedance, Z_S and Z_L which are typically 50 Ohms, but these can be any reasonable value. The matching networks transform the resonator impedances to source and load impedance. These are, typically, simple two element L circuits, the same as you might design on a Smith Chart to get from 150 Ohms to 50 Ohms for example.

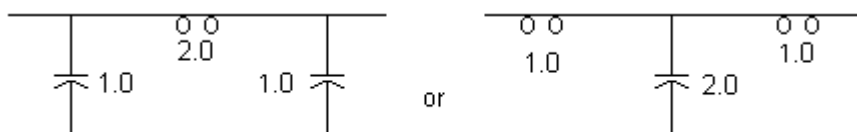
Between the resonator sections are the inverters. The basic idea is to choose a circuit that gives a 90° phase shift, of which there are many, and then choose component values to set the impedance to $\sqrt{Z_1 \cdot Z_2}$. The simplest example of an inverter is a 90° transmission line with a characteristic impedance equal to $\sqrt{Z_1 \cdot Z_2}$.

Topology for a 3 Pole Direct Coupled Filter



A Simple Design Example

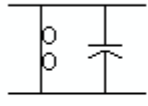
We use a simple design example to show the basic steps used in this design approach. Our first step is to select a prototype filter, such as the 3 pole Butterworth shown here.



We can start with either of these prototypes because we are only interested in using the component values to set the characteristic impedance of the resonator sections. A Direct Coupled Filter has a resonator section for each element in the prototype.

There are many types of resonators that can be used, and we describe several in detail in the resonator section of this paper. To begin with however, we start with LC resonators. In these equations, Z_k is the characteristic impedance of the k_{th} tank circuit and P_k is the k_{th} element value from the prototype circuit. $\omega = 2 \cdot \pi \cdot F_c$, the filter's center frequency.

The characteristic impedance for a parallel and series tank, which can be derived from either component.

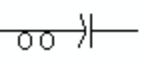


Resonator k

$$Z_k := Q \cdot P_k \cdot \omega \cdot L = \frac{Q \cdot P_k}{\omega \cdot C}$$

$$\omega = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{F_c}{BW}$$



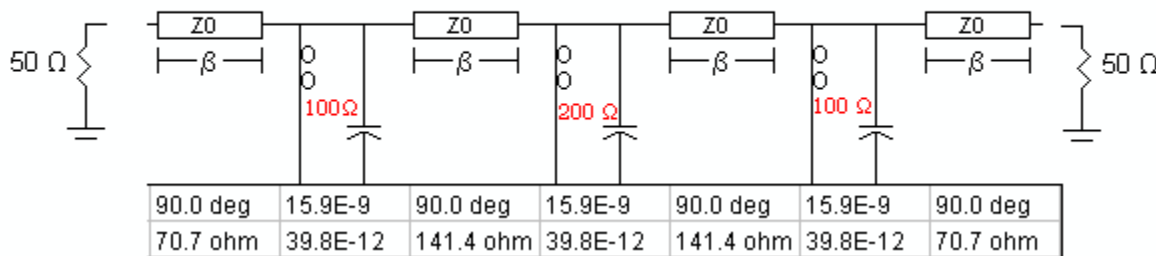
$$Z_k := \frac{\omega \cdot L}{Q \cdot P_k} = \frac{1}{\omega \cdot C \cdot Q \cdot P_k}$$

To start, we select a “nice” inductor value. A “nice” value is typically an off the shelf value that will set Z_k to a value that minimizes the filter's losses. We then calculate an initial value for C, which will set the resonant frequency of the tank circuit to ω . It isn't obvious what value of Z_k will minimize the filter's losses, hence the need for a program that allows you to iterate the on the inductor value (and hence Z_k), and re-simulate to show the filter's losses.

Next, we need an inverter. There are many types of inverters possible, but as a way to demonstrate what these inverter circuits need to achieve, we use quarter wave transmission lines as an example. As you know, a quarter wave transmission line can be used to match between Z_1 and Z_2 by setting the impedance of the line to $\sqrt{Z_1 \cdot Z_2}$. Of course, nobody would use this topology because of the physical length of the line, but this simple circuit is valid and demonstrates the basic design procedure.

This filter is centered at 200 MHz and we set its bandwidth to 40 MHz for a Q of 5. We arbitrarily set the 3 tank impedances to 100, 200, and 100 Ohms. Note, we would normally calculate the tank impedances based on the inductor or capacitor value we want to use, but here we set the tank impedances to convenient values just to show what the inverter circuit does. It should also be noted that we could have set the characteristic impedance of two outer resonators to 50 Ohms, which would have negated the need for the $\frac{1}{4}$ wave lines at the two ends.

An Example Using 1/4 Wave TLines for the Inverters and Matching Circuits The LC Tank Impedances are 100, 200, and 100 Ohms Respectively



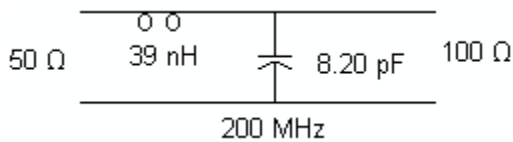
The impedance of the two $\frac{1}{4}$ wave lines used for the matching networks are $\sqrt{100 \cdot 50} = 70.7$ while the impedance of the inverters are $\sqrt{100 \cdot 200} = 141.4$

Using the equations from above for a parallel tank, we show the impedance calculation for the first resonator.

$$Q = 4 \quad P = 1 \quad L = 15.9 \text{ nH} \quad C = 39.8 \text{ pF}$$

$$Z = Q \cdot P \cdot \omega \cdot L \quad Z = 100.028 \quad \omega = (L \cdot C)^{-\frac{1}{2}} = 200 \cdot 10^6 \cdot 2 \cdot \pi$$

Lastly, we want to show an alternative matching circuit. We used $\frac{1}{4}$ wave transmission lines above as a simple example, but we want to make it clear that the matching circuits on the two ends are trivial. If you know how to design a matching circuit on a Smith Chart, then you can easily derive this simple circuit, which we can use in place of the $\frac{1}{4}$ wave lines we have on both ends.



The design process can be summarized as follows:

1. Select a prototype filter, such as a 3 pole Butterworth, for the element values.
2. Choose a tank circuit type.
3. Set one of the element values, typically the inductor, and calculate the resonator's impedance value.
4. Calculate the other resonator value so $\omega = 1/\sqrt{LC}$.
5. Use the resonator impedance values to create an inverter circuit.
6. Design a matching circuit for the two ends.
7. Simulate and iterate through steps 3 – 6 to achieve the lowest insertion loss.

This example was meant to show the basics of this very simple design method.

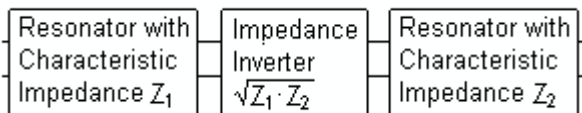
The real power of this methodology lies in the fact that there are many types of resonator, inverter, and matching circuits that can be mixed and matched to create a filter with a wide variety of responses.

In the next three sections, we show how to design several different types of resonator, inverter, and matching circuits.

Inverters

If you have some basic RF design experience, you understand the basic principles involved when a quarter wave transmission line is used as an impedance matching element between Z_1 and Z_2 . Simply set the line impedance $Z = \sqrt{Z_1 \cdot Z_2}$.

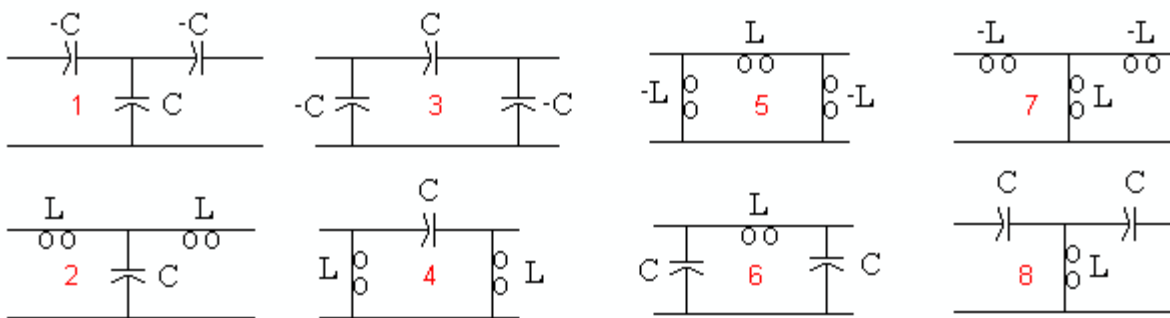
This basic idea is shown here where we have two resonator circuits with an inverter circuit between them.



In his paper, Cohn described how the odd numbered circuits shown below (all of which have negative component values) can be used as impedance inverters where L and C are calculated such that:

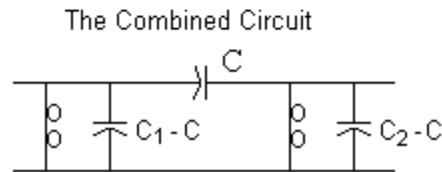
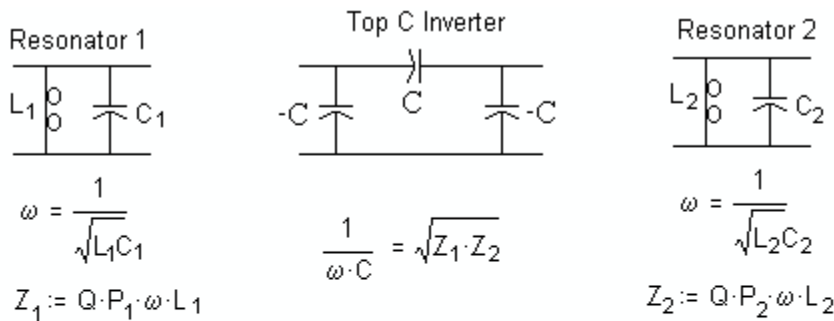
$$\omega \cdot L = \sqrt{Z_1 \cdot Z_2} \quad \text{or} \quad \frac{1}{\omega \cdot C} = \sqrt{Z_1 \cdot Z_2}$$

LC Inverter Circuits



The basic idea is to choose an inverter topology where the two outer components can be absorbed into the resonator structure. With this in mind, it should be clear that circuits 1, 2, 7, and 8 are used with series resonators and circuits 3, 4, 5, and 6 are used with parallel resonant structures.

The next thing to consider is: which element in the resonator do you want to hold constant. Let's consider a common example (as shown below) where we want to use a parallel resonant LC structure and want the final design to have some specific inductor value. Then we need to use inverter circuits 3 or 6 because these 2 circuits allow us to combine the inverter's capacitor into the resonator capacitor. (If using circuit 3, the resonator capacitor must be larger than the inverter's capacitor to avoid a negative combined result, which is seldom a problem.)



Note: Q, P₁, P₂, L₁, L₂, and ω, are known, so we can calculate Z₁ and Z₂ and then calculate the inverter's C value.

If we had wanted to hold the resonator's capacitor to a specific value, then we would have chosen a convenient capacitor value for the resonator, calculated the resonator's inductor value for resonance at ω, and then used either circuit 4 or 5.

Example Calculations for a Top C 3 Pole Filter (a screen shot from Math CAD).

Note that for the sake of simplicity, we use simple, but adequate definitions for Q and BW in this example.

See <http://iowahills.com/Smith%20Chart/MatchingCircuitEquations.html> for impedance matching formulas.

$$F := 200 \cdot 10^6 \quad BW := 40 \cdot 10^6 \quad \omega := 2 \cdot \pi \cdot F \quad Q := \frac{F}{BW}$$

$$RS := 50 \quad RL := 50 \quad \text{RS and RL are the source and load impedances.}$$

$$k := 1..3$$

$$P_k := 0 \quad L_k := 0$$

$$P_1 := 1 \quad P_2 := 2 \quad P_3 := 1 \quad \text{These are the 3 Pole Butterworth low pass prototype values.}$$

$$L_1 := 33 \cdot 10^{-9} \quad L_2 := 47 \cdot 10^{-9} \quad L_3 := 33 \cdot 10^{-9} \quad \text{Set the 3 tank inductors to our desired values.}$$

$$C_k := \frac{1}{\omega^2 \cdot L_k} \quad \text{Calculate the initial value for the tank capacitor.} \quad +$$

We will add the Inverter and Matching C values to these.

$$Z_k := Q \cdot P_k \cdot \omega \cdot L_k \quad \text{Calculate the Characteristic Impedance of the 3 tanks based on the inductor values we are using.}$$

$$\text{InverterC}_{12} := \frac{1}{\sqrt{Z_1 \cdot Z_2} \cdot \omega} \quad \text{InverterC}_{23} := \frac{1}{\sqrt{Z_2 \cdot Z_3} \cdot \omega} \quad \text{Calculate the capacitor inverter values that go between tanks 1 and 2 and tanks 2 and 3.}$$

$$C = \begin{bmatrix} 0 \\ 1.918962 \cdot 10^{-11} \\ 1.347356 \cdot 10^{-11} \\ 1.918962 \cdot 10^{-11} \end{bmatrix} \quad \text{InverterC}_{12} = 2.273994 \cdot 10^{-12}$$

$$\text{InverterC}_{23} = 2.273994 \cdot 10^{-12} \quad Z = \begin{bmatrix} 0 \\ 207.345115 \\ 590.619419 \\ 207.345115 \end{bmatrix}$$

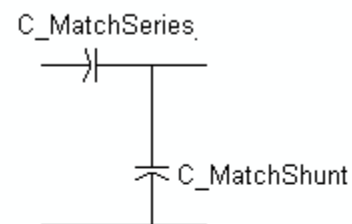
$$C_{\text{MatchSeries_In}} := \frac{1}{RS \cdot \omega \cdot \sqrt{\frac{Z_1}{RS} - 1}}$$

$$C_{\text{MatchSeries_Out}} := \frac{1}{RL \cdot \omega \cdot \sqrt{\frac{Z_3}{RL} - 1}}$$

Calculate the 2 matching circuit C values for the ends.

$$C_{\text{MatchShunt_In}} := \frac{\sqrt{\frac{Z_1}{RS} - 1}}{\omega \cdot Z_1}$$

$$C_{\text{MatchShunt_Out}} := \frac{\sqrt{\frac{Z_3}{RL} - 1}}{\omega \cdot Z_3}$$



$$C_{\text{MatchSeries_In}} = 8.971778 \cdot 10^{-12}$$

$$C_{\text{MatchSeries_Out}} = 8.971778 \cdot 10^{-12}$$

$$C_{\text{MatchShunt_In}} = 6.808289 \cdot 10^{-12}$$

$$C_{\text{MatchShunt_Out}} = 6.808289 \cdot 10^{-12}$$

+

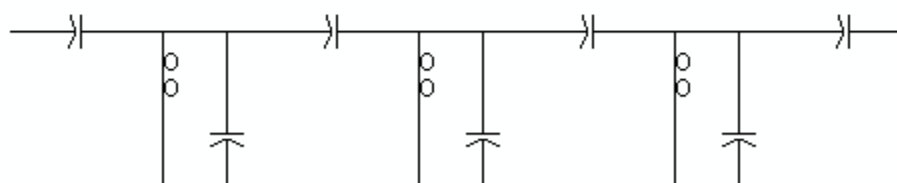
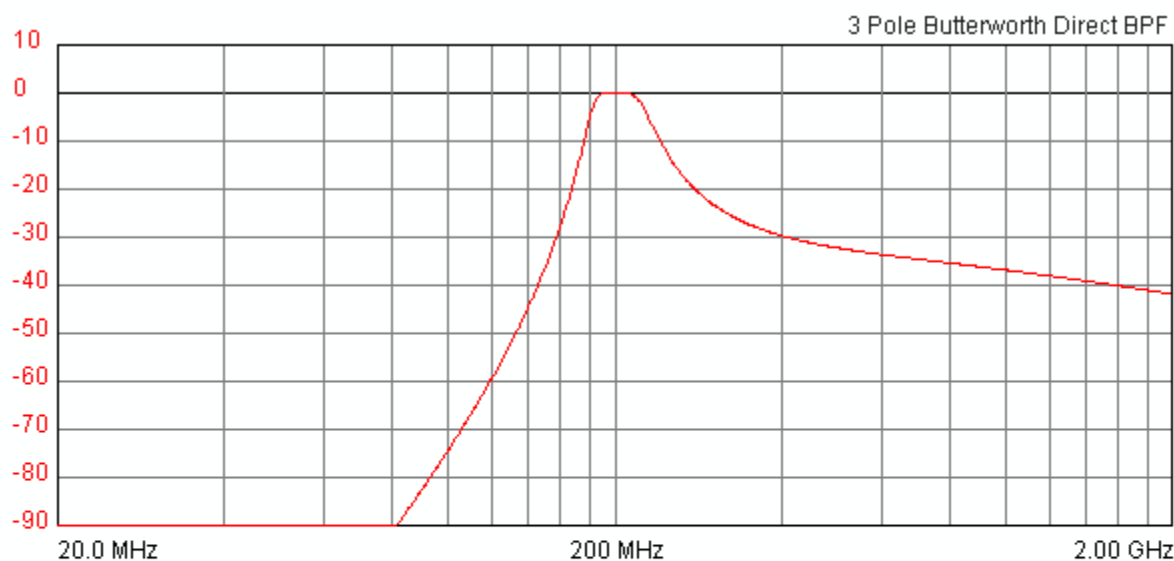
$$C_1 := C_1 - C_{\text{MatchShunt_In}} - \text{InverterC}_{12}$$

Lastly, combine the various capacitor values.

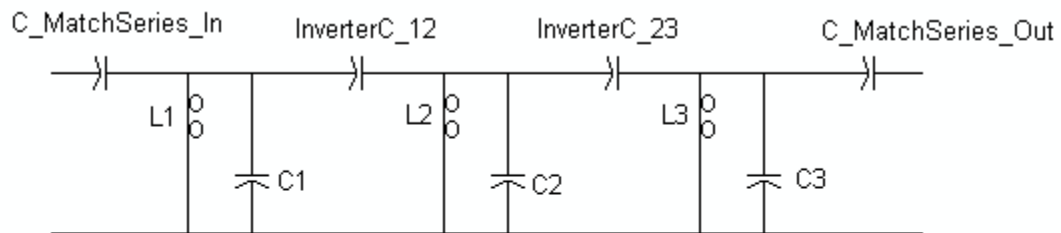
$$C_2 := C_2 - \text{InverterC}_{12} - \text{InverterC}_{23}$$

$$C_3 := C_3 - C_{\text{MatchShunt_Out}} - \text{InverterC}_{23}$$

$$C = \begin{bmatrix} 0 \\ 1.010734 \cdot 10^{-11} \\ 8.925573 \cdot 10^{-12} \\ 1.010734 \cdot 10^{-11} \end{bmatrix}$$



Ind		33.0E-9		47.0E-9		33.0E-9	
Cap	8.97E-12	10.1E-12	2.27E-12	8.93E-12	2.27E-12	10.1E-12	8.97E-12



$$C_1 := C_1 - C_MatchShunt_In - InverterC_12$$

$$C_2 := C_2 - InverterC_12 - InverterC_23$$

$$C_3 := C_3 - C_MatchShunt_Out - InverterC_23$$